Lösungen

Thema: **Wechselstromtechnik**

Scheitelwert und Augenblickswert sinusförmiger Wechselspannungen und Wechselströme

1. $u_1 = \hat{u} \cdot \sin \alpha_{G1} = 34\,V \cdot \sin 30° = \mathbf{17\,V}$; $\quad \mathbf{u_2 = -24\,V}$

2. $\hat{u} = \frac{u}{\sin \alpha_G} = \frac{20{,}8\,V}{\sin 60°} = \mathbf{24\,V}$

3. a) $\hat{u} = \sqrt{2} \cdot 12\,V = \mathbf{17\,V}$ $\qquad$ **b)** $u = \hat{u} \cdot \sin \alpha_B = 17\,V \cdot \sin 0{,}75\pi = \mathbf{12\,V}$

4. a) $T = \frac{1}{f} = \frac{1}{50\,\frac{1}{s}} = 20\,ms$; $\quad \frac{\alpha_B}{2\pi} = \frac{t}{T} \Rightarrow \boldsymbol{\alpha_B} = \frac{2\pi \cdot t}{T} = \frac{2\pi \cdot 4\,ms}{20\,ms} = \mathbf{1{,}26}$

$\frac{\alpha_G}{360°} = \frac{t}{T} \Rightarrow \boldsymbol{\alpha_G} = \frac{t \cdot 360°}{T} = \frac{4\,ms \cdot 360°}{20\,ms} = \mathbf{72°}$

5. a) $u = \hat{u} \cdot \sin \alpha = 412\,V \cdot \sin 15° = 412\,V \cdot 0{,}259 = \mathbf{107\,V}$

b) $u = \hat{u} \cdot \sin \alpha = 412\,V \cdot \sin 72° = 412\,V \cdot 0{,}95 = \mathbf{392\,V}$

c) $u = 412\,V \cdot \sin 331° = 412\,V \cdot (-0{,}485) = \mathbf{-200\,V}$

d) $u = \hat{u} \cdot \sin \alpha = \hat{u} \cdot \sin(\pi/2) = \hat{u} \cdot \sin 1{,}57 = 412\,V \cdot 1 = \mathbf{412\,V}$*

e) $u = \hat{u} \cdot \sin \alpha = 412\,V \cdot \sin(3\pi) = 412\,V \cdot 0 = \mathbf{0\,V}$*

f) $u = \hat{u} \cdot \sin \alpha = 412\,V \cdot \sin 120° = \mathbf{357\,V}$ $\qquad$ **g)** $u = 412\,V \cdot \sin 352° = \mathbf{-57{,}3\,V}$

h) $u = \hat{u} \cdot \sin \alpha = 412\,V \cdot \sin(5/6 \cdot \pi) = 412\,V \cdot \sin 2{,}62 = \mathbf{206\,V}$*

i) $u = 412\,V \cdot \sin 170° = \mathbf{71{,}5\,V}$ $\qquad$ **j)** $u = 412\,V \cdot \sin(\pi/3) = 412\,V \cdot 0{,}866 = \mathbf{357\,V}$*

* Elektronischer Taschenrechner auf RAD

6. $i = \hat{i} \cdot \sin \omega t = 5\,mA \cdot \sin(2\pi \cdot 500\,\frac{1}{s} \cdot 0{,}0003\,s) = 5\,mA \cdot \sin 0{,}9425 = \mathbf{4\,mA}$

7. a) aus Kennlinie: $\quad \mathbf{\hat{u} = 8\,V}$, $T = 12\,ms$; $\quad \mathbf{f} = \frac{1}{T} = \frac{1}{12\,ms} = \mathbf{83{,}3\,Hz}$

$u = \hat{u} \cdot \sin \omega t = 8\,V \cdot \sin(2\pi \cdot 83{,}3\,\frac{1}{s} \cdot 0{,}002\,s) = 8\,V \cdot \sin 1{,}047 = \mathbf{6{,}9\,V}$

8. a) Lösungsweg 1:

$i = \hat{i} \cdot \sin \omega t \Rightarrow \hat{i} = \frac{i}{\sin \omega t} = \frac{20\,A}{\sin(2\pi \cdot 50\,\frac{1}{s} \cdot 0{,}002\,s)}$

$\hat{i} = \frac{20\,A}{\sin 0{,}6283} = \frac{20\,A}{0{,}588} = \mathbf{34\,A}$

Lösungsweg 2:

$T = \frac{1}{f} = \frac{1}{50\,\frac{1}{s}} = 20\,ms$; $\quad \frac{\alpha}{t} = \frac{360°}{T} \Rightarrow \alpha = \frac{t \cdot 360°}{T} = \frac{2\,ms \cdot 360°}{20\,ms} = 36°$

$i = \hat{i} \cdot \sin \alpha \Rightarrow \hat{i} = \frac{i}{\sin \alpha} = \frac{20\,A}{\sin 36°} = \mathbf{34\,A}$

b) $\mathbf{I} = \frac{\hat{i}}{\sqrt{2}} = \frac{34\,A}{\sqrt{2}} = \mathbf{24\,V}$

9. **a)** $\sin \alpha_1 = \frac{u_1}{\hat{u}} = \frac{24\ \text{V}}{34\ \text{V}} = 0{,}706 \Rightarrow \alpha_1 = 45°; \quad \alpha_1' = 180° - \alpha_1 = 135°$

$\frac{t_1}{T} = \frac{\alpha_{G1}}{360°} \Rightarrow \mathbf{t_1} = \frac{\alpha_{G1} \cdot T}{360°} = \frac{45° \cdot 10\ \text{ms}}{360°} = \mathbf{1{,}25\ ms}; \quad \mathbf{t_1' = 3{,}75\ ms}$

b) $\sin \alpha_2 = \frac{u_2}{\hat{u}} = \frac{-17\ \text{V}}{34\ \text{V}} = -0{,}5 \Rightarrow -30° \Rightarrow \alpha_2 = 180° + 30° = 210°; \quad \alpha_2' = 330°$

$\mathbf{t_2} = \frac{\alpha_{G2} \cdot T}{360°} = \frac{210° \cdot 10\ \text{ms}}{360°} = \mathbf{5{,}83\ ms}; \quad t_2' = \mathbf{9{,}17\ ms}$

10. $\hat{i} = \frac{i}{\sin \alpha} = \frac{38{,}6\ \text{A}}{\sin 145°} = \mathbf{67{,}3\ A}$

11. **a)** $+12\ \text{V}: \sin \alpha_1 = \frac{u}{\hat{u}} = \frac{12\ \text{V}}{73\ \text{V}} = 0{,}164 \Rightarrow \boldsymbol{\alpha_1} = \mathbf{9{,}5°}; \quad \boldsymbol{\alpha_2} = 180° - 9{,}5° = \mathbf{170{,}5°}$

$-30\ \text{V}: \sin \alpha_1 = \frac{u}{\hat{u}} = \frac{-30\ \text{V}}{73\ \text{V}} = -0{,}411 \Rightarrow \boldsymbol{\alpha_1} = \mathbf{204°}; \quad \boldsymbol{\alpha_2} = 360° - (\alpha_1 - 180°) = \mathbf{336°}$

b) $T = \frac{1}{f} = \frac{1}{1200\ \text{Hz}} = 0{,}833\ \text{ms}$

$+12\ \text{V}: \frac{t}{T} = \frac{\alpha}{360°} \Rightarrow \mathbf{t_1} = \frac{T \cdot \alpha_1}{360°} = \frac{0{,}833\ \text{ms} \cdot 9{,}5°}{360°} = \mathbf{0{,}022\ ms};$

$\mathbf{t_2} = \frac{0{,}833\ \text{ms} \cdot 170{,}5°}{360°} = \mathbf{0{,}395\ ms}$

$-30\ \text{V}: \mathbf{t_1} = \frac{T \cdot \alpha_1}{360°} = \frac{0{,}833\ \text{ms} \cdot 204{,}3°}{360°} = \mathbf{0{,}473\ ms}$

$\mathbf{t_2} = \frac{0{,}833\ \text{ms} \cdot 336°}{360°} = \mathbf{0{,}777\ ms}$

12. **Lösungsweg 1:**

$u = \hat{u} \cdot \sin \omega t \Rightarrow \sin \omega t = \frac{u}{\hat{u}} = \frac{321\ \text{V}}{707\ \text{V}} = 0{,}454 \Rightarrow \omega t = 0{,}4713$

$\omega t = 2 \cdot \pi \cdot f \cdot t \Rightarrow \mathbf{f} = \frac{\omega \cdot t}{2\pi \cdot t} = \frac{0{,}4713}{2\pi \cdot 0{,}0045\ \text{s}} = \mathbf{16{,}7\ Hz}$

Lösungsweg 2:

$u = \hat{u} \cdot \sin \alpha \Rightarrow \sin \alpha = \frac{u}{\hat{u}} = 0{,}454 \Rightarrow \alpha = 27°$

$\frac{\alpha}{t} = \frac{360°}{T} \Rightarrow T = \frac{360° \cdot t}{\alpha} = \frac{360° \cdot 4{,}5\ \text{ms}}{27°} = 60\ \text{ms} \Rightarrow \mathbf{f} = \frac{1}{T} = \frac{1}{0{,}060\ \text{s}} = \mathbf{16{,}7\ Hz}$

13. **a)** Aus Oszillogramm: $\mathbf{\hat{u}} = 4\ \text{div} \cdot 5\text{V/div} = \mathbf{20\ V}$

b) Aus Oszillogramm: $t_1 = 1\ \text{ms}: \mathbf{u_1} = 2{,}4\ \text{div} \cdot 5\ \text{V/div} = \mathbf{12\ V}$

$t_2 = 8\ \text{ms}: \mathbf{u_2} = 3{,}8\ \text{div} \cdot (-5\ \text{V/div}) = \mathbf{-19\ V}$

c) Aus Oszillogramm: $T = 10\ \text{ms} \Rightarrow f = \frac{1}{T} = \frac{1}{10\ \text{ms}} = 100\ \text{Hz}$

$\mathbf{u_1} = \hat{u} \cdot \sin \omega t_1 = 20\ \text{V} \cdot \sin (2\pi \cdot 100\ \frac{1}{\text{s}} \cdot 0{,}001\ \text{s}) = 20\ \text{V} \cdot \sin 0{,}628 = \mathbf{11{,}8\ V}$

$\mathbf{u_2} = \hat{u} \cdot \sin \omega t_2 = 20\ \text{V} \cdot \sin 5{,}03 = \mathbf{-19\ V}$

d) $\frac{\alpha_{G1}}{360°} = \frac{t_1}{T} \Rightarrow \boldsymbol{\alpha_{G1}} = \frac{t_1 \cdot 360°}{T} = \frac{1\ \text{ms} \cdot 360°}{10\ \text{ms}} = \mathbf{36°}; \quad \boldsymbol{\alpha_{G2}} = \mathbf{288°}$

$\frac{\alpha_{B1}}{2\pi} = \frac{t_1}{T} \Rightarrow \boldsymbol{\alpha_{B1}} = \frac{2\pi \cdot t_1}{T} = \frac{2\pi \cdot 1\ \text{ms}}{10\ \text{ms}} = \mathbf{0{,}628}; \quad \boldsymbol{\alpha_{B2}} = \mathbf{5{,}03}$

Addition sinusförmiger Wechselgrößen gleicher Frequenz

1. a) $\hat{u}_2 = \frac{\hat{u}_1}{2} = \frac{325\ \text{V}}{2} = 162{,}5\ \text{V}$

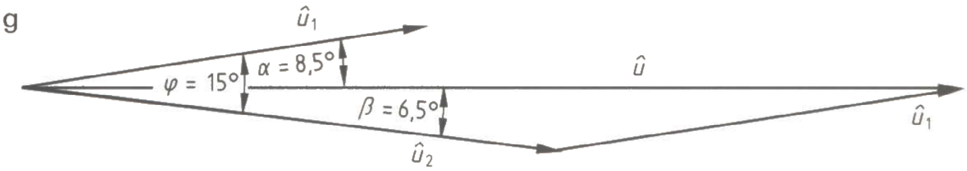
$\hat{u} = \sqrt{\hat{u}_1^2 + \hat{u}_2^2 - 2 \cdot \hat{u}_1 \cdot \hat{u}_2 \cdot \cos(180° - \varphi)}$

$\hat{u} = \sqrt{(325\ \text{V})^2 + (162{,}5\ \text{V})^2 - 2 \cdot 325\ \text{V} \cdot 162{,}5\ \text{V} \cdot \cos(180° - 75°)}$

$\mathbf{\hat{u}} = \sqrt{105625\ \text{V}^2 + 26406\ \text{V}^2 - 105625\ \text{V}^2 \cdot (-0{,}259)} = \mathbf{399\ V}$

b) $\mathbf{U_1} = \frac{\hat{u}_1}{\sqrt{2}} = \mathbf{230\ V}$; $\mathbf{U_2} = \frac{U_1}{2} = \mathbf{115\ V}$; $U = \frac{\hat{u}}{\sqrt{2}}$; $\mathbf{U} = \frac{399\ \text{V}}{\sqrt{2}} = \mathbf{282\ V}$

2. a) Zeichnerische Lösung
Maßstab:
$10\ \text{V} \mathrel{\hat{=}} 0{,}5\ \text{cm}$
$\hat{u}_1 = 84\ \text{V} \mathrel{\hat{=}} 4{,}2\ \text{cm}$
$\hat{u}_2 = 112\ \text{V} \mathrel{\hat{=}} 5{,}6\ \text{cm}$
$\varphi = 15°$
$\hat{u} \mathrel{\hat{=}} 9{,}7\ \text{cm} \Rightarrow \mathbf{\hat{u} = 194\ V}$



zu 102/2.a)

b) $\tan\alpha = \frac{\hat{u}_2 \cdot \sin\varphi}{\hat{u}_1 + \hat{u}_2 \cdot \cos\varphi} = \frac{112\ \text{V} \cdot \sin 15°}{84\ \text{V} + 112\ \text{V} \cdot \cos 15°} = 0{,}151$; $\boldsymbol{\alpha} = \mathbf{8{,}6°}$

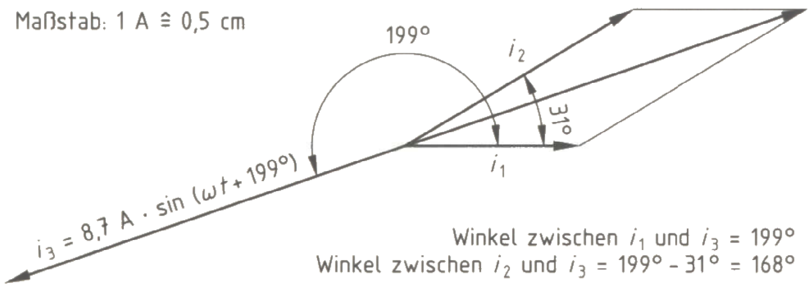
$\tan\beta = \frac{\hat{u}_1 \cdot \sin\varphi}{\hat{u}_2 + \hat{u}_1 \cdot \cos\varphi} = \frac{84\ \text{V} \cdot \sin 15°}{112\ \text{V} + 84\ \text{V} \cdot \cos 15°} = 0{,}113$; $\boldsymbol{\beta} = \mathbf{6{,}4°}$

c) $\hat{u} = \sqrt{\hat{u}_1^2 + \hat{u}_2^2 - 2\hat{u}_1 \cdot \hat{u}_2 \cdot \cos 180° - \varphi}$

$\hat{u} = \sqrt{(84\ \text{V})^2 + (112\ \text{V})^2 - 2 \cdot 84\ \text{V} \cdot 112\ \text{V} \cdot \cos(180° - 15°)}$

$\mathbf{\hat{u}} = \sqrt{7056\ \text{V}^2 + 12544\ \text{V}^2 - 18816 \cdot (-0{,}966)\ \text{V}^2} = \mathbf{194\ V}$

3. a) Maßstab: $1\ \text{A} \mathrel{\hat{=}} 0{,}5\ \text{cm}$



zu 102/3.a)

b) $\mathbf{\hat{i}_3 = 8{,}7\ A}$

4. a) $\tan\beta = \frac{\hat{i}_2 \cdot \sin\varphi}{\hat{i}_1 + \hat{i}_2 \cdot \cos\varphi} = \frac{3{,}5\ \text{A} \cdot \sin 30°}{2{,}5\ \text{A} + 3{,}5\ \text{A} \cdot \cos 30°} = 0{,}3164 \Rightarrow \beta = 17{,}56°$

$\hat{i}_{1,2} = \sqrt{(3{,}5\ \text{A})^2 + (2{,}5\ \text{A})^2 - 2 \cdot 3{,}5\ \text{A} \cdot 2{,}5\ \text{A} \cdot \cos(180° - 30°)} = \sqrt{33{,}6\ \text{A}^2} = 5{,}8\ \text{A}$

$\tan\beta' = \frac{\hat{i}_3 \cdot \sin\varphi'}{\hat{i}_{1,2} + \hat{i}_3 \cdot \cos\varphi'} = \frac{\hat{i}_3 \cdot \sin(75° - \beta)}{\hat{i}_{1,2} + \hat{i}_3 \cdot \cos(75° - \beta)}$

$\tan\beta' = \frac{4{,}5\ \text{A} \cdot \sin(75° - 17{,}56°)}{5{,}8\ \text{A} + 4{,}5\ \text{A} \cdot \cos(75° - 17{,}56°)} = 0{,}461 \Rightarrow \beta' = 24{,}7°$

$\gamma = \beta + \beta' = 17{,}56° + 24{,}7° = \mathbf{42{,}3°}$; **Winkel zwischen i_1 und i_2 = 30°**
Winkel zwischen i_1 und i_3 = 75° **Winkel zwischen i_1 und i_4 = 222,3°**

b) $\hat{i}_4 = \sqrt{\hat{i}_{1,2} + \hat{i}_3^2 - 2 \cdot \hat{i}_{1,2} \cdot \hat{i}_3 \cdot \cos(180° - (75° - \beta))}$

$\hat{i}_4 = \sqrt{(5{,}8\ \text{A})^2 + (4{,}5\ \text{A})^2 - 2 \cdot 5{,}8\ \text{A} \cdot 4{,}5\ \text{A} \cdot \cos 122{,}6°} = \mathbf{9{,}06\ A}$

Wechselstromkreis mit Wirkwiderständen

1. a) $\mathbf{U_w} = \sqrt{P \cdot R} = \sqrt{2\,W \cdot 680\,\Omega} = \mathbf{36{,}9\,V}$; b) $\mathbf{\hat{u}_w} = \sqrt{2} \cdot U_w = \sqrt{2} \cdot 36{,}9\,V = \mathbf{52{,}2\,V}$

2. $\mathbf{R_1} = \frac{U_w^2}{P_1} = \frac{230^2\,V^2}{950\,W} = \mathbf{55{,}7\,\Omega}$; $\mathbf{R_2} = \frac{U_w^2}{P_2} = \frac{230^2\,V^2}{1\,700\,W} = \mathbf{31{,}1\,\Omega}$

3. a) $\mathbf{R} = \frac{U_w^2}{P} = \frac{230^2\,V^2}{1\,200\,W} = \mathbf{44{,}1\,\Omega}$

b) $I_w = \frac{P}{U_w} = \frac{1\,200\,W}{230\,V} = 5{,}22\,A$; $\mathbf{\hat{i}_w} = \sqrt{2} \cdot I_w = \sqrt{2} \cdot 5{,}22\,A = \mathbf{7{,}38\,A}$

c) $\mathbf{\hat{p}} = 2 \cdot P = 2 \cdot 1\,200\,W = \mathbf{2\,400\,W}$

4. a) $\mathbf{\hat{p}} = 2 \cdot P = 2 \cdot 900\,W = \mathbf{1\,800\,W}$

b) $I_w = \frac{\hat{i}_w}{\sqrt{2}} = \frac{5{,}53\,A}{\sqrt{2}} = 3{,}91\,A$; $\mathbf{U_w} = \frac{P}{I_w} = \frac{900\,W}{3{,}91\,A} = \mathbf{230\,V}$

5. a) Aus Liniendiagramm (Bild 3):

$\hat{p} \mathrel{\hat{=}} 20\,mm \Rightarrow \hat{p} = 2\,000\,W$; $\hat{i}_w \mathrel{\hat{=}} 5\,mm \Rightarrow \hat{i}_w = 6\,A$

b) $P = \frac{1}{2} \cdot \hat{p} = \frac{1}{2} \cdot 2\,000\,W = 1\,000\,W$; $I_w = \frac{\hat{i}_w}{\sqrt{2}} = \frac{6\,A}{\sqrt{2}} = 4{,}24\,A$

$\mathbf{R} = \frac{P}{I_w^2} = \frac{1\,000\,W}{(4{,}24\,A)^2} = \mathbf{55{,}6\,\Omega}$

$\mathbf{U_w} = I_w \cdot R = 4{,}24\,A \cdot 55{,}6\,\Omega = \mathbf{235\,V}$

6. a) $\mathbf{\hat{p}} = \hat{u}_w \cdot \hat{i}_w = 325\,V \cdot 6{,}15\,A = \mathbf{2\,kW}$ b) $\mathbf{P} = \frac{1}{2} \cdot \hat{p} = \frac{1}{2} \cdot 2\,000\,W = \mathbf{1\,kW}$

c) **Lösungsweg 1:**

$u_w = \hat{u}_w \cdot \sin 30° = 325\,V \cdot 0{,}5 = 162{,}5\,V$; $i_w = \hat{i}_w \cdot \sin 30° = 6{,}15\,A \cdot 0{,}5 = 3{,}1\,A$

$\mathbf{p} = u_w \cdot i_w = 162{,}5\,V \cdot 3{,}1\,A = \mathbf{500\,W}$

Lösungsweg 2:

$p = \hat{u}_w \cdot \sin 30° \cdot \hat{i} \cdot \sin 30° = \hat{u} \cdot \hat{i} \cdot \sin^2 30° = \hat{p} \cdot \sin^2 30°$

$\mathbf{\hat{p}} = 2\,000\,W \cdot \sin^2 30° = 2\,000\,W \cdot \sin^2 0{,}5 = 2\,000 \cdot 0{,}25 = \mathbf{500\,W}$

7. a) $\mathbf{P} = \frac{U^2}{R} = \frac{230^2\,V^2}{17{,}6\,\Omega} = \mathbf{3\,kW}$

b) **Lösungsweg 1:**

$\hat{u}_w = \sqrt{2} \cdot U_w = \sqrt{2} \cdot 230\,V = 325\,V$; $\hat{i}_w = \frac{\hat{u}_w}{R} = \frac{325\,V}{17{,}6\,\Omega} = 18{,}5\,A$

t = 1 ms: $p = u_w \cdot i_w = \hat{u}_w \sin(\omega t) \cdot \hat{i}_w \sin(\omega t) = \hat{u}_w \cdot \hat{i}_w \cdot \sin^2(\omega t)$

$p = 325\,V \cdot 18{,}5\,A \cdot \sin^2(2\pi \cdot 50\,\frac{1}{s} \cdot 0{,}001\,s) = 6{,}01\,kVA \cdot \sin^2 0{,}314$

$\mathbf{p} = 6{,}01\,kVA \cdot 0{,}0955 = \mathbf{574\,W}$

t = 2,5 ms: $\mathbf{p = 3\,kW}$; t = 4 ms: $\mathbf{p = 5{,}43\,kW}$

Lösungsweg 2:

$\hat{u}_w = \sqrt{2} \cdot u_w = \sqrt{2} \cdot 230\,V = 325\,V$; $u_w = \hat{u}_w \cdot \sin(\omega t) = 325 \sin(2\pi \cdot 50\,\frac{1}{s} \cdot 0{,}001\,s) = 100{,}4\,V$

$\mathbf{p} = \frac{U_w^2}{R} = \frac{100{,}4^2\,V^2}{17{,}6\,\Omega} = \mathbf{573\,W}$

t = 2,5 ms: $\mathbf{p = 3{,}0\,kW}$; t = 4 ms: $\mathbf{p = 5{,}43\,kW}$